Ranking Aggregation for Meta-Search Engine

Chunheng Jiang

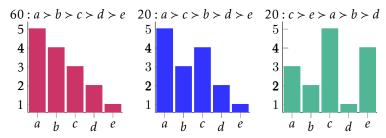
Computer Science Department, RPI

Ranking aggregation aims to produce a ranking list $\pi \in \Pi(\mathcal{C})$ from the preference profile P of n voters. Different aggregation methods (or voting rules) may produce different preference rankings.

- 1. Candidates: $C = \{c_1, c_2, \dots, c_m\}$
- 2. Ranking Space: $\Pi(\mathcal{C}) = \{\pi_1, \pi_2, \ldots\}$
- 3. Profile: $P = \{\sigma_1, \sigma_2, \dots, \sigma_n\}, \ \sigma_i \in \Pi(\mathcal{C}), 1 \le i \le n$

The optimal Kemeny ranking is commonly used to evaluate the performance of an aggregation method, and we expect the combined result π^* is or at least very close to the optimal Kemeny ranking over P, i.e.

$$\pi^* pprox_{d_{KL}} \operatorname*{arg\,min} d_{KL}(\pi, P)$$
 $\pi \in \Pi(\mathcal{C})$



Positions	Margin	Condorcet	Borda
$\pi_1(a) = 1, \pi_1(b) = 2$	1		$s_a - s_b = 1$
$\pi_2(a) = 1, \pi_2(b) = 3$	2>1	a > b	$s_a - s_b = 2$
$\pi_3(a) = 3, \pi_3(b) = 4$	1<1		$s_a - s_b = 1$

	0.6	0.2	0.2
а	1	1	3
b	2	3	4
С	3	2	1
d	4	4	5
е	5	5	2

- 1. a > b: 1/2, 1/3, 3/5
- 2. a > c: 1/3, 1/2, 3/1

Important Factors:

- 1. Positional information
- 2. Positional difference
- 3. Unequal ballot-weights

- 1. Positional score based methods, e.g. Borda count
- 2. Kemeny optimal aggregation
- 3. Probabilistic models
- 4. Machine learning based methods

Pairwise Margin Rule

$$s_{\pi}(i,j) = \frac{\pi(j) - \pi(i)}{\min\{\pi(i), \pi(j)\}}, \forall c_i, c_j \in \mathcal{C},$$

where $\pi(i)$ is the position of c_i in π . When c_i is ranked first, $\pi(i) = 1$; when c_i has no place at all, $\pi(i) = |\pi| + 1$.

The aggregated score for each individual candidate is a weighted sum over all preference rankings:

$$s_i = \sum_{\pi \in \Pi(\mathcal{C})} w_{\pi} s_i(\pi) = \sum_{\pi \in \Pi(\mathcal{C})} \sum_{1 \leq j \leq n} w_{\pi} \frac{\pi(j) - \pi(i)}{\min\{\pi(i), \pi(j)\}}, 1 \leq i \leq n,$$

where $\sum_{\pi \in \Pi(\mathcal{C})} w_{\pi} = 1$ and $0 \le w_{\pi} \le 1$.

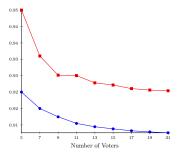
Pairwise Margin Rule

▶ Monotonically increasing w.r.t $\pi(i)$, i.e.

$$\forall c_i \succ_{\pi} c_j, s_i(\pi) > s_j(\pi)$$

▶ If $m \ge 3$, no positional scoring rule satisfies the Condorcet consistency. How about making it satisfying the criterion as nearly as possible?

YES. Better satisfiability to Condorcet consistency than Borda rule when m = 3.

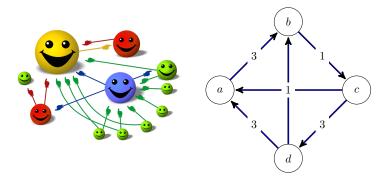


Link-based Voting Rule

$$S_i \leftarrow (1-d)/N + d \sum_{j \to i} S_j/N_j$$

where S_i is the PageRank score of page i (or candidate c_i in voting case), it can be computed based on two factors: the importance of other pages (e.g. j) which have hyperlinks to i and the total number N_j of hyperlinks they have. Page j's PageRank score will be divided into N_j parts equally, and $j \rightarrow i$ indicates that page j refers to i, or has the hyperlink of page i, and d is the probability of visiting pages via hyperlinks, and 1-d is the probability of random walk.

Link-based Voting Rule



The weighted majority graph in voting can be used to model a weighted PageRank surfing method, and produce a preference ranking iteratively.

Link-based Scoring Rule

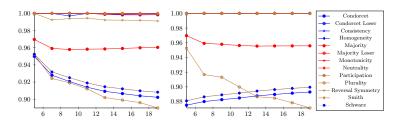
Weighted PageRank model:

$$\begin{cases} P_i = \sum_{j=1}^n P_j \cdot Pr(c_i|c_j) \\ Pr(c_i|c_j) = \sigma(\sum_{\pi \in \Pi(\mathcal{C})} s_{\pi}(i,j)) / \left[\sum_{k=1}^n \sigma(\sum_{\pi \in \Pi(\mathcal{C})} s_{\pi}(k,j))\right] \end{cases}$$

Scores are distributed based on the probability of c_i is preferred to c_j , which is computed with both absolute and relative (margin) positional information. We get the scores by solving P = AP iteratively, where $a_{ij} = Pr(c_i|c_j)$ and $\sum_i a_{ij} = 1$, $\sigma(x) = 1/[1 + e^{-x}]$, A is a stochastic matrix.

Link-based Scoring Rule

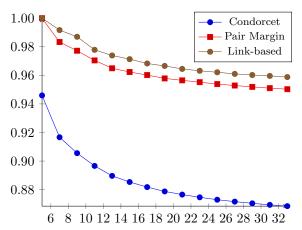
Link-based Scoring Rule (L) v.s. Borda Rule (R) w.r.t their satisfiabilities to popular fairness criteria:



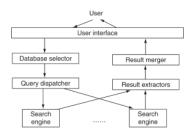
Good news and Bad news: Condorcet Criterion

Link-based Scoring Rule

Link-based Scoring Rule, Pair Margin Rule v.s. Condorcet Method w.r.t their similarities to Borda Rule:



Meta Search Engine



Crawler -> Indexer -> Searcher -> Ranker (ranking aggregation comes into play)

- ► Individual Search Engines: Google, Bing, Yahoo, Ask, Baidu and Blekko.
- ► Aggregated: first SERPs
- Pros: diverse topics, flexible demonstration, and no distractions from Ads.
- ► Cons: no independent data base, and long waiting time.

Ask	Baidu	Bing	Blekko	Google
✓ Yahoo	MS.Academic	G.Scholar		☐ All/None
omputatio	nal social choice			Search
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